

Sequences and Series

4

LEARNING OBJECTIVES

In this chapter, you learn how to use GC to

- ⇒ Generate a sequence using `seq ()`.
- ⇒ Find the Sum of a Sequence using `sum ()` or `cumSum ()`.
- ⇒ Find the Sum to Infinity of a GP.
- ⇒ Find the n th term of a Recurring Sequence using `SEO` mode.
- ⇒ Solve Quadratic Equation using `PlySmt2` application.
- ⇒ Evaluate \sum .
- ⇒ Find the Root(s) of an Equation under `GRAPH` mode.
- ⇒ Determine the Behaviour of a Sequence.

Example 4.1

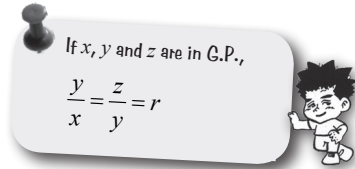
A geometric series has common ratio r , and an arithmetic series has first term a and common difference d , where a and d are non-zero. The first three terms of the geometric series are equal to the first, sixth and tenth terms respectively of the arithmetic series.

- (i) Show that $5r^2 - 9r + 4 = 0$.
- (ii) Deduce that the geometric series is convergent and find, in terms of a , the sum to infinity.
- (iii) The sum of the first n terms of the geometric series is denoted by S . Given that $a > 0$, find the least value of n for which S exceeds 99% of the sum to infinity.

SOLUTION

$$(i) \quad r = \frac{a + (6-1)d}{a} = \frac{a + (10-1)d}{a + (6-1)d}$$

$$r = 1 + 5\left(\frac{d}{a}\right) = \frac{1 + 9\left(\frac{d}{a}\right)}{1 + 5\left(\frac{d}{a}\right)}$$



$$\therefore \frac{d}{a} = \frac{r-1}{5} \Rightarrow r = \frac{1 + 9\left(\frac{r-1}{5}\right)}{1 + 5\left(\frac{r-1}{5}\right)} \Rightarrow r^2 = 1 + 9\left(\frac{r-1}{5}\right) \Rightarrow 5r^2 = 5 + 9r - 9$$

$$\Rightarrow 5r^2 - 9r + 4 = 0 \text{ (shown)}$$

$$(ii) \quad 5r^2 - 9r + 4 = 0 \Rightarrow (r-1)(5r-4) = 0 \Rightarrow r = 1 \text{ or } r = \frac{4}{5}$$

Since $d \neq 0$, the three terms of GP are not the same, thus $r \neq 1$.

Hence, $r = \frac{4}{5} \Rightarrow$ Since $|r| < 1$, the geometric series is convergent.

$$\text{And the sum to infinity} = \frac{a}{1 - \frac{4}{5}} = 5a.$$

$$(iii) \quad S_n > 0.99S_\infty$$

$$\frac{a(1-0.8^n)}{1-0.8} > 0.99 \frac{a}{1-0.8}$$

$$1 - 0.8^n > 0.99$$

$$0.8^n < 0.01$$

$$n \lg 0.8 < \lg 0.01$$

$$n > \frac{\lg 0.01}{\lg 0.8}$$

$$n > 20.6$$

Hence, the least value of n is 21.

(You may use GC to verify your answer.)

2nd STAT ▶▶

5

```

NAMES OPS MATH
1:min(
2:max(
3:mean(
4:median(
5:sum(
6:Prod(
7↓stdDev(

```

Select LIST, MATH, sum(.

Note that sum (LIST).

2nd STAT ▶▶

5

```

NAMES OPS MATH
1:SortA(
2:SortD(
3:dim(
4:Fill(
5:seq(
6:cumSum(
7↓List(

```

Select LIST, OPS, seq(.

Note that seq (expression, variable, begin value, end value).

[Complete the rest of the expression as shown.]

```

sum(seq(0.8^X,X,
1,200))

```

4

Answer: "4" which is where $a = 0.8$ and $r = 0.8$. (Here, we assume a takes a positive value.)

STAT 1

▲

[Type in the expression as shown.]

L1	L2	L3	1
-----	-----	-----	
L1 =seq(X,X,1,50			

Select STAT, EDIT, Edit.

Go up to the title L1.

Generate a list of integers from 1 to 50 under L1.

▶▲

[Type in the expression as shown.]

L1	L2	L3	2
1 2 3 4 5 6	-----	-----	
L2 =0.8^L1			

Go up to the title L2.

Generate a list of values of 0.8^{L1} under L2.



[Type in the expression as shown.]

▢ ... until a value $> 4 \times 0.99$, i.e. 3.96, is reached.

L1	L2	L3	3
1	.8	-----	
2	.64		
3	.512		
4	.4096		
5	.32768		
6	.26214		
7	.20972		
L3=cumSum(L2			

NAMES	OPS	MATH
1:SortA(
2:SortD(
3:dim(
4:Fill(
5:seq(
6:cumSum(
7:List(

L1	L2	L3	3
15	.03518	3.8593	
16	.02815	3.8874	
17	.02252	3.9099	
18	.01801	3.9279	
19	.01441	3.9424	
20	.01153	3.9539	
21	.00922	3.9631	
L3(21)=3.96310651...			

Go up to the title L3.

Generate a list of cumulative values of L2 under L3.

For cumSum, press $\text{2nd}[\text{STAT}][\text{D}][6]$,

i.e. LIST, OPS, cumSum(.

Note that the syntax of cumSum is cumSum(LIST).

\therefore We obtain the least value of n is 21.

Example 4.2

- (i) Patrick saves \$20 on 1 January 2008. On the first day of each subsequent month he saves \$4 more than in the previous month, so that he saves \$24 on 1 February 2008, \$28 on 1 March 2009, and so on. On what date will he first have saved over \$5000 in total?
- (ii) Kenny puts \$20 on 1 January 2008 into a bank account which pays compound interest at a rate of 3% per month on the last day of each month. He puts a further \$20 into the account on the first day of each subsequent month.
- (a) How much compound interest has his original \$20 earned at the end of 3 years?
- (b) How much in total, correct to the nearest dollar, is in the account at the end of 3 years?
- (c) After how many complete months will the total in the account first exceed \$5000?

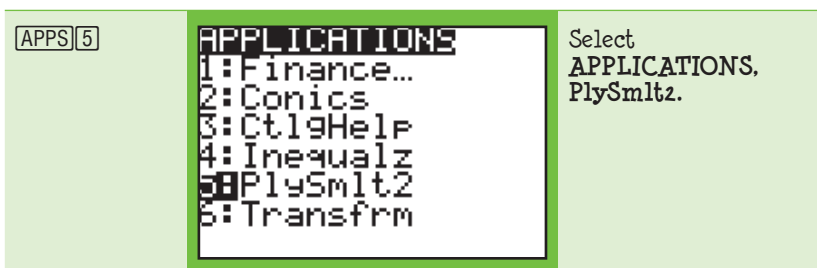
SOLUTION

(i) $T_1 = 20, d = 4$

$$\frac{n}{2}[2(20) + 4(n-1)] > 5000 \Rightarrow n^2 + 9n - 2500 > 0 \Rightarrow n < -54.7 \text{ or } n > 45.7$$

\therefore Hence, Patrick will first have saved over \$5000 in total on 1 October 2011.

(You may use GC to find the roots.)



The image shows a TI-84 Plus calculator screen. In the top left corner, 'APPS' is displayed in a box with the number '5' next to it. The main screen shows a list of applications: '1: Finance...', '2: Conics', '3: Ct19Help', '4: Inequalz', '5: PlySmlt2', and '6: Transfrm'. The '5: PlySmlt2' option is highlighted with a white background. To the right of the screen, the text 'Select APPLICATIONS, PlySmlt2.' is visible.

1

```

MAIN MENU
1: POLY ROOT FINDER
2: SIMULT EQN SOLVER
3: ABOUT
4: POLY HELP
5: SIMULT HELP
6: QUIT POLYSMLT

```

Select POLY ROOT FINDER.

GRAPH

```

POLY ROOT FINDER MODE
ORDER  1 2 3 4 5 6 7 8 9 10
REAL   a+bi  re^(θi)
DEC    FRAC
NORMAL SCI ENG
FLOAT  0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
MAIN   (HELP) (NEXT)

```

Select NEXT.

[Enter the coefficients accordingly.]

```

a2x²+a1x+a0=0
a2 = 1
a1 = 9
a0 = -2500
MAIN MODE CLR LOAD SOLVE

```

GRAPH

```

a2x²+a1x+a0=0
x1 = -54.70209159
x2 = 45.70209159
MAIN MODE COEF STD F4/D

```

Select SOLVE.


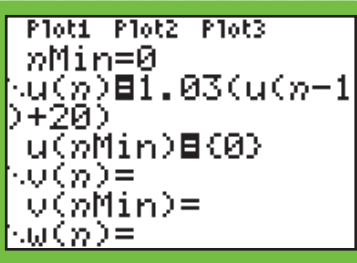
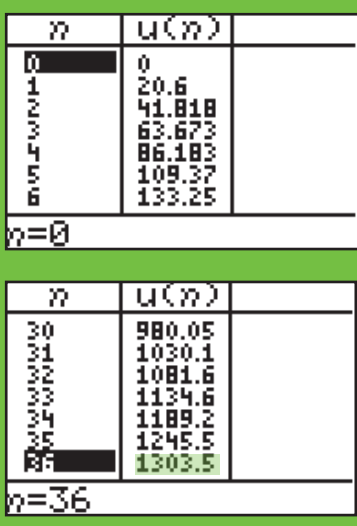
Answers: 45.7 or -54.7

- (ii) (a) The required compound interest = $20(1.03)^{36} - 20 = \$38.0$ (3 s.f.)
 (b) The required amount = $20(1.03) + 20(1.03)^2 + \dots + 20(1.03)^{36}$

$$= \frac{20(1.03)[(1.03)^{36} - 1]}{1.03 - 1}$$

$$= \$1303$$

(You may use GC to verify your answer.)

<p>MODE ▾ ▾ ▾ ▾ ▾ ▾ ▾ ▾ ENTER</p>		<p>Enter MODE settings. Select SEQ.</p>
<p>[Enter the data as shown.]</p>		<p>Press 2nd7 for u and X,T,θ,n for n.</p>
<p>2ndGRAPH ▸ ... until n = 36 is reached.</p>		<p>Select TABLE. $\therefore u(36) = 1303.5$ ✓</p>

$$(c) 20(1.03) + 20(1.03)^2 + \dots + 20(1.03)^n > 5000$$

$$\frac{20(1.03)(1.03^n - 1)}{1.03 - 1} > 5000$$

$$1.03^n > 8.282$$

$$n > \frac{\lg 8.282}{\lg 1.03}$$

$$n > 71.5$$

The amount will first exceed \$5000 after 72 months.

(You may use GC to verify your answer.)

▶ ... until $u(n)$ first reaches 5000 or above.	n	$u(n)$		
	66	4144		
	67	4288.9		
	68	4438.1		
	69	4591.9		
	70	4750.2		
	71	4913.3		
	72	5081.3		
	$n=72$			

Note that
 $u(71) = 4913.3 < 5000$
 $u(72) = 5081.3 > 5000$
 $\therefore n = 72. \checkmark$

Example 4.3

A sequence u_1, u_2, u_3, \dots is such that $u_1 = 1$ and

$$u_{n+1} = u_n - \frac{3n^2 + 3n + 1}{n^3(n+1)^3}, \text{ for all } n \geq 1.$$

(i) Use the method of mathematical induction to prove that $u_n = \frac{1}{n^3}$.

(ii) Hence find $\sum_{n=1}^N \frac{3n^2 + 3n + 1}{n^3(n+1)^3}$.

(iii) Give a reason why the series in part (ii) is convergent and state the sum to infinity.

(iv) Use your answer to part (ii) to find $\sum_{n=2}^N \frac{3n^2 - 3n + 1}{n^3(n-1)^3}$.

SOLUTION

- (i) Let P_n be the statement $u_n = \frac{1}{n^3}$ where $u_{n+1} = u_n - \frac{3n^2 + 3n + 1}{n^3(n+1)^3}$ and $u_1 = 1$ for $n \in \mathbb{Z}^+$.

$$\text{When } n=1, \text{ L.H.S.} = u_1 = 1; \text{ R.H.S.} = \frac{1}{1^2} = 1.$$

\therefore L.H.S. = R.H.S.

$\therefore P_1$ is true.

Assume P_k is true for some $k \in \mathbb{Z}^+$, i.e. $u_k = \frac{1}{k^3}$.

To prove P_{k+1} is true, i.e. $u_{k+1} = \frac{1}{(k+1)^3}$

$$\begin{aligned} \text{L.H.S.} &= u_{k+1} = u_k - \frac{3k^2 + 3k + 1}{k^3(k+1)^3} \\ &= \frac{1}{k^3} - \frac{3k^2 + 3k + 1}{k^3(k+1)^3} \\ &= \frac{(k+1)^3 - 3k^2 - 3k - 1}{k^3(k+1)^3} \\ &= \frac{k^3 + 3k^2 + 3k + 1 - 3k^2 - 3k - 1}{k^3(k+1)^3} \\ &= \frac{1}{(k+1)^3} \\ &= \text{R.H.S.} \end{aligned}$$

$\therefore P_{k+1}$ is true whenever P_k is true.

Since P_1 is true and $P_k \Rightarrow P_{k+1}$ is true by Mathematical Induction, P_n is true for all $n \in \mathbb{Z}^+$.

$$\begin{aligned} \text{(ii)} \quad \sum_{n=1}^N \frac{3n^2 + 3n + 1}{n^3(n+1)^3} &= \sum_{n=1}^N (u_n - u_{n+1}) \\ &= \cancel{u_1} - \cancel{u_2} \\ &\quad + \cancel{u_2} - \cancel{u_3} \\ &\quad \dots \\ &\quad + \cancel{u_{N-1}} - \cancel{u_N} \\ &\quad + u_N - u_{N+1} \\ &= u_1 - u_{N+1} \\ &= 1 - \frac{1}{(N+1)^3} \end{aligned}$$

(You may use GC to verify your answer.)

<p>[STAT] 1</p> <p>▲</p> <p>[Type in the expression as shown.]</p>	<table border="1"> <thead> <tr> <th>L1</th> <th>L2</th> <th>L3</th> <th>1</th> </tr> </thead> <tbody> <tr> <td>-----</td> <td>-----</td> <td>-----</td> <td></td> </tr> <tr> <td colspan="4">L1 = seq(X, X, 1, 8)</td> </tr> </tbody> </table>	L1	L2	L3	1	-----	-----	-----		L1 = seq(X, X, 1, 8)				<p>Select STAT, EDIT, Edit.</p> <p>Go up to the title L1.</p> <p>Generate a list of integers from 1 to 8 under L1.</p>																								
L1	L2	L3	1																																			
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L1 = seq(X, X, 1, 8)																																						
<p>▶▲</p> <p>Type in L2</p> $= \frac{3L_1^2 + 3L_1 + 1}{L_1^3 (L_1 + L)^3}$	<table border="1"> <thead> <tr> <th>L1</th> <th>L2</th> <th>L3</th> <th>2</th> </tr> </thead> <tbody> <tr> <td>-----</td> <td>-----</td> <td>-----</td> <td></td> </tr> <tr> <td colspan="4">L2 = ...^3(L1+1)^3</td> </tr> </tbody> </table>	L1	L2	L3	2	-----	-----	-----		L2 = ...^3(L1+1)^3				<p>Go up to the title L2.</p> <p>Generate a list of values based on the given formula under L2.</p>																								
L1	L2	L3	2																																			
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L2 = ...^3(L1+1)^3																																						
<p>▶▲</p> <p>[Type in the expression as shown.]</p>	<table border="1"> <thead> <tr> <th>L1</th> <th>L2</th> <th>L3</th> <th>3</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>.875</td> <td>-----</td> <td></td> </tr> <tr> <td>2</td> <td>.08796</td> <td></td> <td></td> </tr> <tr> <td>3</td> <td>.02141</td> <td></td> <td></td> </tr> <tr> <td>4</td> <td>.00763</td> <td></td> <td></td> </tr> <tr> <td>5</td> <td>.00337</td> <td></td> <td></td> </tr> <tr> <td>6</td> <td>.00171</td> <td></td> <td></td> </tr> <tr> <td>7</td> <td>9.6E-4</td> <td></td> <td></td> </tr> <tr> <td colspan="4">L3 = cumSum(L2)</td> </tr> </tbody> </table>	L1	L2	L3	3	1	.875	-----		2	.08796			3	.02141			4	.00763			5	.00337			6	.00171			7	9.6E-4			L3 = cumSum(L2)				<p>Go up to the title L3.</p> <p>Generate a list of cumulative values of L2 under L3.</p>
L1	L2	L3	3																																			
1	.875	-----																																				
2	.08796																																					
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<p>▶ ... until L1=8 is reached.</p>	<table border="1"> <thead> <tr> <th>L1</th> <th>L2</th> <th>L3</th> <th>3</th> </tr> </thead> <tbody> <tr> <td>2</td> <td>.08796</td> <td>.96296</td> <td></td> </tr> <tr> <td>3</td> <td>.02141</td> <td>.98438</td> <td></td> </tr> <tr> <td>4</td> <td>.00763</td> <td>.992</td> <td></td> </tr> <tr> <td>5</td> <td>.00337</td> <td>.99537</td> <td></td> </tr> <tr> <td>6</td> <td>.00171</td> <td>.99708</td> <td></td> </tr> <tr> <td>7</td> <td>9.6E-4</td> <td>.99805</td> <td></td> </tr> <tr> <td>8</td> <td>5.8E-4</td> <td>.998628257...</td> <td></td> </tr> <tr> <td colspan="4">L3(8) = .998628257...</td> </tr> </tbody> </table>	L1	L2	L3	3	2	.08796	.96296		3	.02141	.98438		4	.00763	.992		5	.00337	.99537		6	.00171	.99708		7	9.6E-4	.99805		8	5.8E-4	.998628257...		L3(8) = .998628257...				
L1	L2	L3	3																																			
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6	.00171	.99708																																				
7	9.6E-4	.99805																																				
8	5.8E-4	.998628257...																																				
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<p>2nd [MODE]</p>	<table border="1"> <tbody> <tr> <td>1-1/9^3</td> <td>.9986282579</td> </tr> </tbody> </table>	1-1/9^3	.9986282579	<p>Evaluate the answer in part (ii) when $N = 8$.</p> <p>Note that the 2 values are the same. Hence, the expression obtained in part (ii) is verified. ✓</p>																																		
1-1/9^3	.9986282579																																					

(iii) When $N \rightarrow \infty$, $\frac{1}{(N+1)^3} \rightarrow 0 \Rightarrow 1 - \frac{1}{(N+1)^3} \rightarrow 1 \Rightarrow \sum_{n=1}^N \frac{3n^2 + 3n + 1}{n^3(n+1)^3} \rightarrow 1$

Hence, the series in part (ii) is convergent and the sum to infinity is 1.

$$\begin{aligned}
 \text{(iv)} \quad \sum_{n=2}^N \frac{3n^2 - 3n + 1}{n^3(n-1)^3} &= \sum_{r=1}^{N-1} \frac{3(r+1)^2 - 3(r+1) + 1}{(r+1)^3 r^3} \text{ when } n = r + 1 \\
 &= \sum_{r=1}^{N-1} \frac{3(r+1)^2 - 3(r+1) + 1}{r^3(r+1)^3} \\
 &= \sum_{r=1}^{N-1} \frac{3r^2 + 6r + 3 - 3r - 3 + 1}{r^3(r+1)^3} \\
 &= \sum_{r=1}^{N-1} \frac{3r^2 + 3r + 1}{r^3(r+1)^3} \\
 &= 1 - \frac{1}{(N-1+1)^3} \\
 &= 1 - \frac{1}{N^3}
 \end{aligned}$$

(You may use GC again to verify your answer.)

[Apply similar keystrokes shown in part (ii).]

L1	L2	L3	Z
2	-----	-----	
3			
4			
5			
6			
7			
8			
L2 = ..^3(L1-1)^3)			

L1	L2	L3	Z
2	.875	-----	
3	.08796		
4	.02141		
5	.00763		
6	.00337		
7	.00171		
8	9.6E-4		
L3 = cumSum(L2			

Note that L1 now starts from 2.

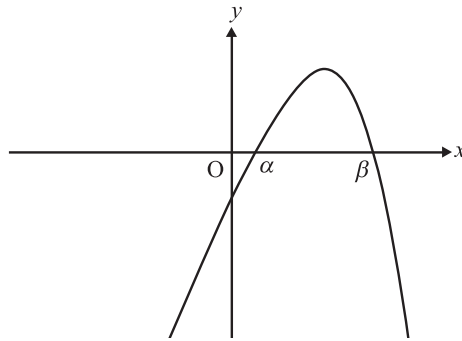
L1	L2	L3	3
2	.875	.875	
3	.08796	.96296	
4	.02141	.98438	
5	.00763	.992	
6	.00337	.99537	
7	.00171	.99708	
8	9.6E-4	.99804	
L3(7) = .998046875			

$1 - 1/8^3$
 .998046875

Evaluate the answer in part (ii) when $N = 8$.

Note that the 2 values are the same. Hence, the expression obtained in part (ii) is verified. ✓

Example 4.4



The diagram shows the graph of $y = 2x - e^{\frac{x}{2}}$. The two roots of the equation are denoted by α and β , where $\alpha < \beta$.

- (i) Find the values of α and β , each correct to 3 decimal places.

A sequence of real numbers x_1, x_2, x_3, \dots satisfies the recurrence relation

$$x_{n+1} = \frac{1}{2}e^{\frac{x_n}{2}}$$

for $n \geq 1$.

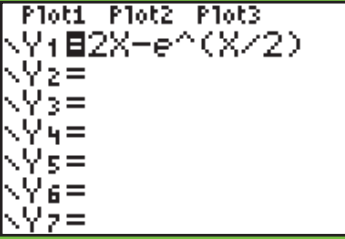
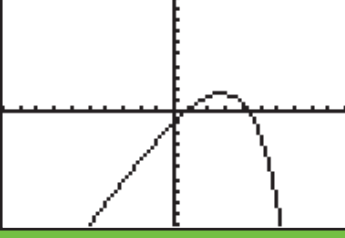
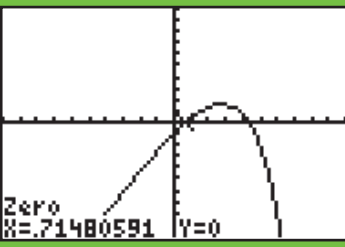
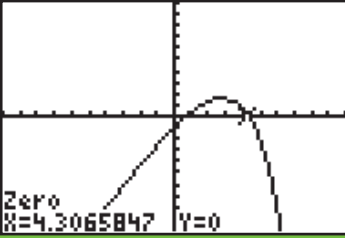
- (ii) Prove algebraically that, if the sequence converges, then it converges to either α or β .
- (iii) Use a calculator to determine the behaviours of the sequence for each of the cases $x_1 = 0$, $x_1 = 3$, $x_1 = 6$.
- (iv) By considering $x_{n+1} - x_n$, prove that

$$x_{n+1} < x_n \text{ if } \alpha < x_n < \beta,$$

$$x_{n+1} > x_n \text{ if } x_n < \alpha \text{ or } x_n > \beta$$

- (v) State briefly how the results in part (iv) relate to the behaviours determined in part (iii).

SOLUTION

<p>Y=</p> <p>[Enter the equation as shown.]</p>		
<p>GRAPH</p>		
<p>2nd TRACE</p> <p>2</p> <p>[Choose the point as shown in the graph.]</p>		<p>Select CALC. Select zero.</p> <p>$\therefore \alpha = 0.715$ ↻</p>
<p>[Apply similar steps for the 2nd root.]</p>		<p>$\therefore \alpha = 4.307$ ↻</p>

(i) From GC, $\alpha = 0.715$ and $\beta = 4.307$.

(ii) $x_{n+1} = \frac{1}{2}e^{\frac{x_n}{2}}$

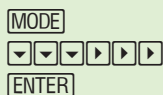
$\Rightarrow 2x_{n+1} = e^{\frac{x_n}{2}}$

$\Rightarrow 2x_{n+1} - e^{\frac{x_n}{2}} = 0$

$\Rightarrow 2L - e^{\frac{L}{2}} = 0$ given that $x_n \rightarrow L$ and $x_{n+1} \rightarrow L$ when $n \rightarrow \infty$.

Since α and β are the roots of $2x - e^{\frac{x}{2}} = 0$, hence x_n converges to α or β if the sequence converges.

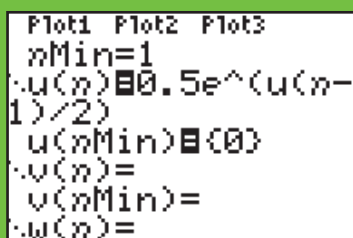
(iii)



Enter MODE settings.

Select SEQ.

[Enter the data as shown.]



Set $u_1 = 0$.



n	$u(n)$	
0	ERROR	
1	0	
2	.5	
3	.64201	
4	.68926	
5	.70573	
6	.71157	

$n=0$

Select TABLE.

▶ ...

[Observe the change in u_n .]

n	$u(n)$	
14	.71481	
15	.71481	
16	.71481	
17	.71481	
18	.71481	
19	.71481	
20	.71481	

$n=20$

Note that the sequence converges to 0.71481 which is a when $x_1 = 0$. ✓

Repeat the same process for the cases $x_1 = 3$ and $x_1 = 6$.

[Apply similar keystrokes except setting $u_1 = 3$.]

```
Plot1 Plot2 Plot3
nMin=1
:u(n)≡0.5e^(u(n-1)/2)
u(nMin)≡(3)
:u(n)=
u(nMin)=
:u(n)=
```

n	$u(n)$	
0	ERROR	
1	3	
2	2.2408	
3	1.5331	
4	1.0762	
5	.85635	
6	.76723	

$n=1$

n	$u(n)$	
14	.71482	
15	.71481	
16	.71481	
17	.71481	
18	.71481	
19	.71481	
20	.71481	

$n=20$

Note that the sequence converges to 0.71481 which is a when $x_1 = 3$. ✓

[Apply similar keystrokes except setting $u_1 = 6$.]

```
Plot1 Plot2 Plot3
nMin=1
:u(n)▣0.5e^(u(n-
1)/2)
u(nMin)▣(6)
:u(n)=
v(nMin)=
:w(n)=
```

n	u(n)
0	ERROR
1	6
2	10.043
3	75.811
4	1.4E16
5	ERROR
6	ERROR

n=0

n	u(n)
6	ERROR
7	ERROR
8	ERROR
9	ERROR
10	ERROR
11	ERROR
12	ERROR

n=12

Note that the sequence diverges when $x_1 = 6$ as shown by the ERROR message. ✓

(iv) $x_{n+1} - x_n = \frac{1}{2}e^{\frac{x_n}{2}} - x_n = \frac{1}{2}e^{\frac{x_n}{2}} - x_n = -\frac{1}{2}(2x_n - e^{\frac{x_n}{2}})$

If $\alpha < x_n < \beta$, $2x_n - e^{\frac{x_n}{2}} > 0 \Rightarrow -\frac{1}{2}(2x_n - e^{\frac{x_n}{2}}) < 0 \Rightarrow x_{n+1} - x_n < 0 \Rightarrow x_{n+1} < x_n$.

If $x_n < \alpha$ or $x_n > \beta$, $2x_n - e^{\frac{x_n}{2}} < 0 \Rightarrow -\frac{1}{2}(2x_n - e^{\frac{x_n}{2}}) > 0 \Rightarrow x_{n+1} - x_n > 0 \Rightarrow x_{n+1} > x_n$.

(v) For $x_1 = 0$ where $x_1 < \alpha \Rightarrow x_{n+1} = \frac{1}{2}e^{\frac{x_n}{2}} - x_n < \frac{1}{2}e^{\frac{\alpha}{2}} = \alpha$, hence $x_1 < x_2 < x_3 < \dots < \alpha$.

For $x_1 = 3$ where $\alpha < x_1 < \beta \Rightarrow x_n > x_{n+1} = \frac{1}{2}e^{\frac{x_n}{2}} - x_n > \frac{1}{2}e^{\frac{\beta}{2}} = \alpha$, hence $x_1 > x_2 > x_3 > \dots > \alpha$.

If $x_1 = 6$ where $x_1 > \beta \Rightarrow x_n < x_{n+1}$, hence $\beta < x_1 < x_2 < x_3 \dots$.



GC Techniques covered in this chapter

TECHNIQUES	4.1	4.2	4.3	4.4
Generate a sequence using seq () .	<input checked="" type="checkbox"/>		<input checked="" type="checkbox"/>	
Find the Sum of a Sequence using sum () or cumSum () .	<input checked="" type="checkbox"/>		<input checked="" type="checkbox"/>	
Find the Sum to Infinity of a GP .	<input checked="" type="checkbox"/>			
Find the nth term of a Recurring Sequence using SEQ mode.		<input checked="" type="checkbox"/>		
Solve Quadratic Equation using PlySmlt2 application.		<input checked="" type="checkbox"/>		
Evaluate Σ .			<input checked="" type="checkbox"/>	
Find the Root(s) of an Equation under GRAPH mode.				<input checked="" type="checkbox"/>
Determine the Behaviour of a Sequence .				<input checked="" type="checkbox"/>